Math 10A with Professor Stankova
Quiz 9; Wednesday, 10/25/2017
Section #106; Time: 10 AM
GSI name: Roy Zhao **Name:** ________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** The general form of the solution to $\dfrac{dy}{dx} = y$ is $y = e^x + C$.

> **Solution:** It is Ce^x, not $e^x + C$.

2. **TRUE** False In order to show that the integral $0 \leq \displaystyle\int_1^\infty \frac{1}{f(x)}dx$ converges, it suffices to find a function $g(x)$ such that $f(x) \geq g(x)$ on $[1, \infty)$ and show that $\displaystyle\int_1^\infty \frac{1}{g(x)}dx$ converges.

> **Solution:** This is true because if $f(x) \geq g(x)$, then $\frac{1}{f(x)} \leq \frac{1}{g(x)}$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) Suppose that $\dfrac{dy}{dx} = \sin(x)\csc(y)$. Find a solution such that $y(0) = \pi$

> **Solution:** We move things to different sides and get that
>
> $$\frac{dy}{\csc(y)} = \sin(y)dy = \sin(x)dx.$$
>
> Taking the integral of both sides gives
>
> $$-\cos(y) = -\cos(x) + C.$$
>
> Now plugging in $y(0) = \pi$, we have that
>
> $$1 = -\cos(\pi) = -\cos(0) + C = -1 + C$$
>
> and so $C = 2$. Therefore, the equation is
>
> $$\cos(y) = \cos(x) - 2.$$

(b) (3 points) Integrate $\displaystyle\int_0^\infty \frac{2x}{(1+x^2)^2}dx$.

Solution: We have that

$$\int_0^\infty \frac{2x}{(1+x^2)^2}dx = \lim_{t\to\infty}\int_0^t \frac{2x}{(1+x^2)^2}dx = \lim_{t\to\infty}\left.\frac{-1}{1+x^2}\right|_0^t = \lim_{t\to\infty}\frac{-1}{1+t^2} - \frac{-1}{1+0^2} = 0-(-1) = 1.$$

(c) (3 points) Does the integral $\displaystyle\int_0^\infty \frac{2x\sin^2(x)}{(1+x^2)^2+e^{-x}}dx$ converge? Hint: Use the previous part.

Solution: We know that $\sin^2(x) \leq 1$ and $(1+x^2)^2+e^{-x} \geq (1+x^2)^2$ so combining these two gives $\dfrac{2x\sin^2(x)}{(1+x^2)^2+e^{-x}} \leq \dfrac{2x}{(1+x^2)^2}$. So, we have that

$$0 \leq \int_0^\infty \frac{2x\sin^2(x)}{(1+x^2)^2+e^{-x}}dx \leq \int_0^\infty \frac{2x}{(1+x^2)^2}dx = 1,$$

and so the integral converges.